The Concept

Our query into combining econometric models has begun with the construction of a high frequency econometric model for Russia, which is a monthly model for short-run quarterly forecasts. We investigate how high frequency models may be combined with low frequency models, with annual observations, for horizons up to 5 to 10 years. We look at the combination of the models, first in general terms, then in terms of the adjustment of their structures and properties to achieve a mutually consistent joint solution, and finally at applications.

High Frequency Model:

$$y_t = Ay_t + A_1 L y_t + A_2 L^2 y_t + \ldots + A_p L^p y_t + Bx_t + e_t$$

y_t = n-column endogenous vector;

x_t = k-column exogenous vector;

$A,\ A_1, \ldots, A_p$ **= (n, n) matrix of coefficients multiplying endogenous variables;**

L = polynomial in the lag operator L: $Ly_t = y_{t-1}$, $L^2 y_t = y_{t-2}, \ldots, L^p y_t = y_{t-p}$;

B = (n, k) matrix of coefficients multiplying exogenous variables;

e_t = n-column vector of error terms.

Low Frequency Model:

$$y_s = Ay_s + A_1 L y_s + A_2 L^2 y_s + \ldots + A_q L^q y_s + Bx_s + e_s$$

(The structure is similar to high frequency, with differences in dimension, time horizon, and lags)

Computing a lagged vector

$$y_s^0 \equiv A_1 y_{s-1} + \ldots + A_q y_{s-q}$$

in an iterative process and merging it into the first column in matrix

$B_S \equiv (b_1, b_2, \ldots, b_r)$ **,**

the transformation allows to obtain the solution in the form:

$\hat{y}_s = A\hat{y}_s + B_s x_s$ **, or** $\hat{y}_s = (I - A)^{-1} B_s x_s$.

Types of Solutions

- an independent solution of the low frequency model
- a combined solution that uses a loss function
- a combined solution by inserting GDP from the high frequency model
- a combined solution based on a targeted growth scenario

Application (Model)

We have built a simplified version of an annual econometric model for Ukraine's economy based on macroeconomic information computed by the Institute of Economic Forecasting of Ukraine's National Academy of Sciences. Along with *GDP*, included are: capital stock *K*, commissioned capital stock *DK*, total investment *I*, average number of workers *L*, and average loan interest rate *RKN*. Annual data for 1993-2003 were used. All monetary variables, *GDP*, *K*, *DK*, and *I*, are in 1996 constant prices.

Using a Loss Function

Minimize GDP_{AN}

subject to

$$GDP_{AN} \geq GDP_{MO};\ GDP_{AN} \geq -27400 + 0.0922\ K + 2999 L$$

$$K \geq 0.9954 K_{-1} + 1.1340 DK;$$

$$DK \geq -8680.7930 + 0.7076\ I$$

$$I \geq -10089.560 + 0.3453 GDP - 24.6160\ RKN$$

$$L \geq 28.2335 - 0.5859\ T + 0.0126\ T^2$$

$$RKN \geq 150.6904 - 5.5866\ T$$

Exact Joint Solution

The high frequency model provides monthly solutions for n endogenous variables with a horizon of one year, and the low frequency model provides annual solutions for m endogenous variables with a horizon of five years. From the high frequency model, the annual GDP is estimated by aggregating monthly GDP values, and is accepted as the first-year GDP in the low frequency model. The procedure reduces the number of endogenous variables in the low frequency model from m to (m-1) for the first year; for the rest of the years it remains m.

Designing Joint Solutions

With the use a loss function or designing an exact joint solution, only the first year is affected. We first find an independent solution for 2004-2008. It helps determine which optimization problem, minimization or maximization, is appropriate for the loss function. Furthermore, we combine mixed frequencies by, first, replacing the 2004 GDP in this independent solution (Table 2) with the solution based on the loss function. Then, we create an exact joint solution by inserting the 2004 value of GDP from the high frequency model into the low frequency model.

Model 1, 2004-2008 Independent Solution

:eqgdp

:eqdk

:eql

:eqk

:eqi

:eqrkn

Assign @all f

Model 2, 2004 Solution Based on the Loss Function

gdp96=gdpLP

:eqdk

:eql

:eqk

:eqi

:eqrkn

Model 3, 2005-2008 Solution with 2004 GDP from the Loss Function

:eqgdp

:eqdk

:eql

Assign @all g

:eqk

:eqi

:eqrkn

Model 4, 2004 Solution with GDP from the High Frequency Model

gdp96=gdpan96 :eqk

:eqdk :eqi

:eql :eqrkn

Targeted Growth Forecast

The designed methodology can be used to produce targeted scenarios of economic growth, with a sustained growth rate compatible with a single-digit inflation rate. Such a scenario may be based on either a loss function or a solution from the high frequency model. We assess a potential impact of a targeted 5% growth rate of GDP on the rest of the variables. With a conservative estimate of a 0.5-1% productivity growth, our decomposition for the factors of production shows that capital stock should increase by 2.5-3% and employment by 2%. The job creation target is realistic in view of lingering high unemployment. The capital stock target calls for a 6-7% growth in investment, and the latter, a steeper interest rate decline than in other forecasts in Table 2.

Model 5, 2005-2008 Solution, Targeted Growth Rates (2004 GDP=High Frequency Model)

```
gdp96=gdp96(-1)*1.05          :eqk
:eqdk                         :eqi
l=l(-1)*1.02
rkn=69.9359-2.2463(t+0.5z)
Assign @all h
```

Table 2. Four Forecasts from the Low Frequency Model
I. Independent Solution of the Low Frequency Model

Year	GDP	K	DK	I	L
2004	90238.43	543501.2	5845.722	20530.31	21.47951
2005	90443.75	547753.1	5935.423	20657.07	21.43481
2006	90713.73	552105.3	6040.934	20806.2	21.41528
2007	91050.38	556575.8	6162.797	20978.44	21.42092
2008	91455.50	561183.1	6325.431	21174.37	21.45173

Table 2. Four Forecasts from the Low Frequency Model II. Solution with 2004 GDP from the Loss Function

Year	GDP	K	DK	I	L
2004	87023.00	542386.1	4862.391	19140.60	21.47951
2005	90349.32	546458.2	5772.451	20426.80	21.43481
2006	90611.41	550694.7	5933.778	20654.82	21.41528
2007	90944.85	555114.5	6112.553	20907.49	21.42092
2008	91351.48	559737.2	6309.214	21185.43	21.45173

Table 2. Four Forecasts from the Low Frequency Model III. Solution with 2004 GDP from High Frequency Model

Year	GDP	K	DK	I	L
2004	120185.10	551818.5	13180.440	30896.51	21.47951
2005	91056.38	556202.5	6085.400	20869.14	21.43481
2006	91332.98	560687.9	6192.553	21020.57	21.41528
2007	91676.59	565293.1	6316.115	21195.21	21.42092
2008	92089.05	570036.4	6456.543	21393.67	21.45173

Table 2. Four Forecasts from the Low Frequency Model IV. Solution with Targeted Growth Rates (2004 GDP=High Frequency Model)

Year	GDP	K	DK	I	L
2004	120185.10	551818.5	13180.44	30896.51	21.47951
2005	126194.30	565984.1	14711.38	33060.19	21.90910
2006	132504.00	581904.6	16315.91	35327.87	22.34729
2007	139129.20	599659.6	17997.71	37704.76	22.79423
2008	146085.70	619322.8	19760.64	40196.32	23.25012

Comparisons

Solution four is most indicative of the performance of Ukraine's economy in the last five years. A scenario built from this solution is consistent with the pattern of growth achieved since 2000, while inertia reflected by other solutions incorporates depressed outputs and rising inflation of the turbulent 1990s. Obtaining this solution required the two models, high and low frequency, working together thus reinforcing the premise of the usefulness of our methodology.

In solutions combining two types of models, high and low frequency, we presume that short-term forecasts are generally more reliable than long-term forecasts. Since the solution in the high frequency model is the sum of monthly forecasts, this model may generally be the best barometer for the first-year solution in the low frequency model. Accordingly, we design procedures for an adjustment of the low frequency model in tune with the high frequency model.

Conclusion and Extension, I

With the availability of plausible high frequency and low frequency forecasts, the methodology provides a consistent way of combining the two types of models based on consistent computation techniques. **To apply the outlined methodology in real-life situations, we must both refine the data used in the construction of the models and design the low frequency model more realistically and in more detail. Applications will be done for the Ukrainian and, then, Russian economies. Besides building reliable forecasts at mixed frequencies and targeted scenarios of sustained economic growth, we shall extend the methodology to incorporate both macro econometric and input-output models.**

Conclusion and Extension, II

We are also going to link our methodology more closely to the macroeconomic decision making of the government of Ukraine. One of the sectors to be incorporated in greater detail is foreign trade, especially at the low frequency level. It should include exports and imports classified by commodity and major trade partners. The expanded coverage of the consolidated government budget is also an important task. Among its first priorities, the new government of Ukraine has envisaged a sharp increase in social spending, such as pensions, payment to would-be mothers, and minimum wages. The government is looking for ways to raise revenues by reducing public investment, new public construction, VAT breaks, etc. We can help by bringing together data from high frequency and low frequency econometric models and IO tables, providing consistent analysis of flows on both demand and supply sides of the reforming economy.